Networking Low-Power Energy Harvesting Devices: Measurements and Algorithms

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Abstract—Recent advances in energy harvesting materials and ultra-low-power communications will soon enable the realization of networks composed of energy harvesting devices. These devices will operate using very low ambient energy, such as energy harvested from indoor lights. We focus on characterizing the light energy availability in indoor environments and on developing energy allocation algorithms for energy harvesting devices. First, we present results of our long-term indoor radiant energy measurements, which provide important inputs required for algorithm and system design (e.g., determining the required battery sizes). Then, we focus on algorithm development, which requires nontraditional approaches, since energy harvesting shifts the nature of energy-aware protocols from minimizing energy expenditure to optimizing it. Moreover, in many cases, different energy storage types (rechargeable battery and a capacitor) require different algorithms. We develop algorithms for calculating time fair energy allocation in systems with deterministic energy inputs, as well as in systems where energy inputs are stochastic.

Index Terms—Energy harvesting, ultra-low-power networking, active RFID, indoor radiant energy, measurements, energy-aware algorithms

1 INTRODUCTION

Recent advances in the areas of solar, piezoelectric, and thermal energy harvesting [40], and in ultra-low-power wireless communications [49] will soon enable the realization of perpetual energy harvesting wireless devices. When networked together, they can compose rechargeable sensor networks [26], [41], [54], networks of computational RFID tags [20], and Energy Harvesting Active Networked Tags (EnHANTS) [15], [18]. Such networks will find applications in various areas, and therefore, the wireless industry is already engaged in the design of various devices (e.g., [5]).

In this paper, we focus on devices that harvest environmental light energy. Since there is a three orders of magnitude difference between the light energy available indoor and outdoor [18], [42], significantly different algorithms are required for different environments. However, there is lack of data and analysis regarding the energy availability in such environments. Hence, over the past two years, we have been conducting a first-of-its-kind measurement campaign that enables characterizing the energy availability in indoor environments. We describe the results and show that they provide insights that can be used for the development of energy-harvesting-aware algorithms and systems.

Clearly, there has been an extensive research effort in the area of energy efficient algorithms for sensor networks and

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for wireless networks in general. However, for devices with renewable energy sources, fundamentally different problems arise. Hence, in the second part of this paper, we focus on developing algorithms for determining the energy spending rates and the data rates in various scenarios.

To describe our contributions, we introduce below several dimensions of the vast algorithm design space for energy-harvesting devices:

- Environmental energy model: deterministic and partially predictable energy profile, stochastic process, and model-free.
- Energy storage type: battery and capacitor.
- Ratio of energy storage capacity to energy harvested: large to small.
- Time granularity: subseconds to days.
- Problem size: stand-alone node, node pair (link), cluster, and multihop network.

The combinations of values along these dimensions induce several “working points,” some of which have been studied recently (see Section 2).

1.1 Environmental Energy Models

The model representing harvested energy depends on various parameters such as the energy source (e.g., solar or kinetic), the properties of the environment, and the device’s behavior (stationary, semistationary, or mobile). Fig. 1 provides examples of radiant (light) energy sources in different settings. In Fig. 1a, the energy availability is time-dependent and predictable. On the other hand, in Fig. 1b that corresponds to an indoor environment, it is time-dependent and periodic, but harder to predict. Time-dependent and somewhat periodic behaviors (along with inputs such as weather forecasts) would allow to develop an energy profile [12], [27]. We will refer to ideal energy profiles that accurately represent the future as deterministic.
Energy behavior that does not warrant a time-dependent profile appears in Fig. 1c, which shows the irradiance recorded by a mobile device carried around Times Square in New York City at nighttime. In this case, the energy can be modeled by a stochastic process. Other scenarios where stochastic models are a good fit are a floorboard that gathers energy when stepped on and a solar cell in a room where lights go on and off as people enter and leave. Finally, in some settings not relying on an energy model (a model-free approach) is most suitable.

1.2 Energy Storage Types—Linear and Nonlinear
To operate when not directly powered by environmental energy, energy harvesting devices need energy storage: a rechargeable battery or a capacitor. Rechargeable batteries can be modeled by an ideal linear model, where the changes in the energy stored are linearly related to the amounts of energy harvested or spent or that recharging opportunities are not missed. On the other hand, with relatively large storage, simpler algorithms can be used.

Time granularity. Nodes can characterize the received energy and make decisions on timescales from seconds to days. This timescale is related to the storage-harvesting ratio and the environmental energy model.

Problem/network size. Energy harvesting affects nodes’ individual decisions, pairwise (link) decisions, and behavior of networked nodes (e.g., routing and rate adaptation).

1.4 Our Contributions
First, we present the results of a 16 month-long indoor radiant energy measurements campaign and a mobile outdoor light energy study that provide important inputs to the design of algorithms. We discuss the energy available in various indoor environments. We also show that in indoor environments, the energy models are mostly partially predictable and that simple parameters can significantly improve predictions when the temporal granularity is at the order of days. The indoor light energy traces that we have collected are available at enhants.ee.columbia.edu and in the CRAWDAD repository [19]. To the best of our knowledge, this work is the first to present long-term indoor radiant energy measurements.

Second, we formulate resource allocation problems for energy-harvesting devices. The energy available to such devices often varies in time (e.g., throughout the day or among different days). Hence, in this paper, we aim to achieve “smooth” allocation of resources along the time axis in the presence of varying environmental energy.

We consider deterministic energy profile and stochastic environmental energy models, for battery-based systems and for capacitor-based systems, and focus on the cases of a single node and a node pair (link). For the deterministic profile environmental energy model, we use the lexicographic maximization and utility maximization frameworks to obtain the energy spending rate allocations for a node and the data rate allocations for a link. For the stochastic environmental energy model, we consider the case in which the energy inputs are i.i.d. random variables (e.g., a mobile device outdoors), and show how to treat it as an average-cost Markov decision process (MDP). We obtain optimal energy spending policies (both for battery-based and capacitor-based systems) for a single node and a node pair (link) that can be precomputed in advance. To the best of our

1.3 Storage Capacity, Decision Timescale, and Problem Size
Storage capacity versus amount of energy harvested. Energy storage capacity can vary from 0.16 J for an EnerChips device [1] to 4,700 J for an AA battery. The environmental energy availability also varies widely, from thousands

of $J/cm^2/day$ in sunny outdoor conditions to under $2 J/cm^2/day$ in indoor environments (see Section 4). Different combinations require different algorithmic approaches. For example, when the storage is small compared to the harvesting rate, the algorithms must continuously keep track of the energy levels, to guarantee that the storage is not depleted or that recharging opportunities are not missed. On the other hand, with relatively large storage, simpler algorithms can be used.

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knowledge, our work is the first that models the nonlinearity of the capacitor-based system illustrated in Fig. 2. We provide numerical results that demonstrate its effect.

This paper is organized as follows: Section 2 reviews the related work. Section 3 presents the model and Section 4 describes the measurements. Sections 5 and 6 describe algorithms for the deterministic profile and stochastic process energy models, respectively. Section 7 presents the numerical results. We summarize and discuss future work in Section 8.

2 RELATED WORK

Energy efficiency in wireless networks has long been a subject of research. In comparison, energy harvesting in wireless networks has only recently started gaining attention. The developments in this area include wireless energy-harvesting device design and development [26], [41], [46], [47], [53], [54], and exploration of theoretic and algorithmic approaches.

In this paper, we characterize indoor light energy for low-power energy-harvesting devices. Since large-scale outdoor solar panels have been used for decades, properties of the Sun's energy were examined in depth [3], [31], [42]. Practical outdoor solar energy considerations for energy-harvesting in sensor networks (e.g., light obstructions and scattering) were discussed in [47]. Until recently using indoor light energy for networking applications was considered impractical, and indoor light was studied mostly in the areas of architecture and ergonomics [21], [44]. However, in these domains, the important factor is how humans perceive the given light (photometric characterization—i.e., measurements in Lux) rather than the energy of the light (radiometric characterization). Photometric measurements by sensor nodes were reported in [2] and [20]. Photometric measurements, however, do not provide energetic characterization, and there is currently lack of data (e.g., traces) and analysis (e.g., variability, predictability, and correlations) regarding energy availability [42].

This paper also deals with resource allocation for energy-harvesting devices. The related work in this area can be classified according to the environmental energy model employed and related to the general settings described in the previous section:

- **Deterministic profile.** In [23] and [27], duty cycle adjustments (mostly for single nodes) are considered. Transmission power adaptation and transmission scheduling for a scenario with an energy-harvesting transmitter and two receivers are examined in [50] and [8], respectively. For a network, various metrics are considered including data collection rates [12], data retrieval rates [51], throughput maximization [11], and routing efficiency [33]. Per-slot short-term predictions are used to obtain (per-slot) data rates in [34].
- **Partially predictable profile.** While considering energy predictable, [11], [27], [34], [38] have provisions for adjustments in cases in which the predictions are inaccurate.
- **Stochastic process.** Dynamic activation of energy-harvesting sensors is described in [25] for a single node, and for a cluster in [28]. Admission and power allocation control policies are developed in [13]. Routing and scheduling policies are developed in [30]. Maximizing the utility of the average data rates via joint power allocation and energy management is examined in [24]. Energy allocation policies for source-channel coding are developed in [10].
- **Model-free approach.** Duty cycle adjustments for a single node (and under the linear storage model) are examined in [48]. A capacitor-based system is presented and the capacitor leakage is studied in [54].

We aim to allocate nodes' resources in a "smooth" way with respect to time. The need for policies that enable such behavior in energy-harvesting devices has been previously noted [12], [27], [37], [48]. Smoothing node duty cycles using a control theory approach is examined in [48]. Energy allocation vectors with minimal variance are sought in [37]. Both [37] and [48] consider linear energy storage models and focus mainly on single node scenarios. We note that the approach introduced in [52] for throughput optimization in QoS-constrained single node scenarios (for non-energy-harvesting devices) can also be used to achieve smooth energy allocation in energy-harvesting devices (where finite energy storage constraint can be related to the QoS buffer constraint [50]). A throughput optimization framework for energy-harvesting nodes [11], developed in parallel with our work, can also be extended to achieve smooth resource allocation. However, applications of these frameworks to energy-harvesting scenarios result in implicit assumptions of linear energy storage. The model developed in this paper allows incorporating general (linear and nonlinear) energy storage models. Furthermore, we formulate problems and present practical algorithms for both single node and link scenarios.

We note that resource allocation in energy harvesting devices has some similarities with power consumption scheduling in power networks (e.g., [32] and references therein). However, these works consider scenarios where energy sources are centralized and infinite. In contrast, in our settings energy availability is restricted, and is specific to each node and each time slot.

3 MODEL AND PRELIMINARIES

In this paper, we focus both on light measurements and on resource allocation problems. The relationships between variables characterizing energy availability are illustrated in Fig. 3. Table 1 summarizes the notation.

We focus on discrete-time models, where the time axis is separated into $K$ slots, and a decision is made at the
beginning of a slot \( i = \{0, 1, \ldots, K - 1\} \). We denote the energy storage capacity by \( C \) and the amount of energy stored by \( B(i) \) \((0 \leq B(i) \leq C)\). We denote the initial and the final energy levels by \( B_0 \) and \( B_K \), respectively.

Our measurements record irradiance, radiant energy incident onto surface (in \( \text{W}/\text{cm}^2 \)), denoted by \( I \). Irradiation \( H_T \) (in \( \text{J}/\text{cm}^2 \)) is the integral of irradiance over a time period \( T \). In characterizing environmental light energy, we are particularly interested in diurnal (daily) environmental energy availability. For \( T = 24 \) hours, we denote the daily irradiation by \( H_D \).

The amount of energy (in \( \text{J} \)) a solar cell with given physical properties (size, efficiency) can harvest in a time slot \( i \) is denoted by \( Q(i) \). For a solar cell with area \( A \) and efficiency \( \eta, D(i) = A \cdot \eta \cdot H(i) \). For the numerical results presented in this paper we use \( A = 10 \text{ cm}^2 \) and \( \eta = 1\% \) (i.e., efficiency of an organic solar cell) [18].

The energy a node harvests from the environment in a time slot \( i \) is denoted by \( Q(i) \). \( Q(i) \) is a function of \( D(i) \), and may also depend on \( B(i) \). Specifically, for a battery-based device, \( Q(i) = D(i) \). For a capacitor-based device, \( Q(i) = q(D(i), B(i)) \), where \( q(D(i), B(i)) \) is a nonlinear function of \( D(i) \) (see Section 1.2). We refer to energy storage where \( Q(i) \) is linear in \( D(i) \) as linear energy storage, and to energy storage where \( Q(i) \) is nonlinear in \( D(i) \) as nonlinear energy storage. Functions \( q(D(i), B(i)) \) for a capacitor, derived from capacitors' electric properties, are shown in Fig. 2. To derive numerical results for nonlinear energy storage, we use \( q(D(i), B(i)) = D(i) - D(i) \cdot (B(i) - C/2)^2/(\beta_{\text{nonlin}} \cdot (C/2)^3) \), where \( \beta_{\text{nonlin}} \) is the energy storage nonlinearity parameter. These functions have properties similar to the functions shown in Fig. 2.

The energy spending rate is denoted by \( s(i) \). The “storage evolution” of energy harvesting devices can be expressed as

\[
B(i) = \min\{B(i - 1) + Q(i - 1) - s(i - 1), C\}. \tag{1}
\]

We denote the total amount of energy the device is allocating by \( \tilde{Q} \), where \( \tilde{Q} = \sum Q(i) + (B_0 - B_K) \). For simplicity, some of the developed energy allocation algorithms use quantized \( B(i) \) and \( Q(i) \) values. We denote the quantization resolution by \( \Delta \).

We consider the behavior of single nodes and node pairs (links). We denote the endpoints of a link by \( u \) and \( v \), and use these as subscripts for link endpoints’ energy variables (e.g., \( C_u \) and \( B_{0,u} \) correspond, respectively, to node \( u \)’s energy storage capacity and initial storage state). We denote the data rates of \( u \) and \( v \) by \( r_u(i) \) and \( r_v(i) \), respectively. For a single node, we optimize the energy spending rates \( s(i) \), which can provide inputs for determining transmission power, duty cycle, sensing rate, or communication rate. For a link, we optimize the communication rates \( r_u(i) \) and \( r_v(i) \). We denote the costs to transmit and receive a bit by \( c_u \) and \( c_v \).

Often the incoming energy varies throughout the day or among different days. We aim to allocate the energy or the data rates as much as possible in a uniform way with respect to time. We achieve this objective by using the lexicographic maximization and utility maximization frameworks. These frameworks are typically applied to achieve fairness among nodes [9], [12], [29], [34], [39]. In this paper, we apply them to achieve time-fair resource allocation.

In the lexicographic maximization framework, we lexicographically maximize the vector \( \{s(0), \ldots, s(K - 1)\} \) (for a node), or the vector \( \{r_u(0), \ldots, r_u(K - 1), r_v(0), \ldots, r_v(K - 1)\} \) (for a link). In utility maximization framework, we maximize \( \sum_{i=0}^{K-1} U(s(i)) \) (for a single node) or \( \sum_{i=0}^{K-1} [U(r_u(i)) + U(r_v(i))] \) (for a link), where \( U(\cdot) \) are concave nondecreasing twice-differentiable continuous functions (e.g., \( U(\cdot) = \log(\cdot), U(\cdot) = \sqrt{\cdot}, U(\cdot) = (\cdot)^{1-\alpha}/(1 - \alpha), \alpha > 1 \)). To derive numerical results, we use \( U(\cdot) = \log(\cdot) \) or \( U(\cdot) = \log(1 + \cdot) \). In general, the solutions obtained by applying the two frameworks are not the same. The solutions are identical in certain cases, such as those examined in Lemma 1 and in Observation 1.

### 4 Characterizing Light Energy

To characterize indoor energy availability, since June 2009 we have been conducting a light measurement study in office buildings in New York City. In this study, we take long-term measurements of irradiance \( I \), in units \( \text{W}/\text{cm}^2 \), in several indoor locations, and also study a set of shorter-term indoor and outdoor mobile device measurements. Table 2 provides a summary of the indoor measurement locations. The locations are shown schematically in Fig. 4. For the measurements, we use TAOS TSL230rd photometric sensors [4] installed on LabJack U3 DAQ devices. These photometric sensors have a high dynamic range, allowing to capture widely varying irradiance conditions. We verified the accuracy of the sensors with a NIST-traceable Newport 818-UV photodetector. In addition to the indoor measurements, we also analyze a set of outdoor irradiance traces provided by the US Department of Energy National Renewable Energy Laboratory (NREL) [3].

The provided measurements and irradiance traces can be used to determine the performance achievable by a particular device, for system design (e.g., choosing a suitable energy storage or energy harvesting system component), and for determining which algorithms to use. The traces we have collected can also be used as energy feeds to simulators and

**Table 1**

| \( K \) | Number of slots |
| \( I \) | Irradiance (\( \text{W}/\text{cm}^2 \)) |
| \( H \) | Irradiation (\( \text{J}/\text{cm}^2 \)) |
| \( D \) | Energy available to a device (\( I \)) |
| \( C \) | Energy storage capacity (\( C \)) |
| \( B_0, B_K \) | Energy storage state, initial, and final levels (\( J \)) |
| \( Q \) | Energy harvested (\( J \)) |
| \( s \) | Energy spending rate (\( \text{J}/\text{slot} \)) |
| \( \tilde{Q} \) | Total energy to be allocated (\( J \)) |
| \( \Delta \) | Quantization resolution (\( J \)) |
| \( r \) | Data rate (bits/s) |
| \( c_u, c_v \) | Energetic costs to transmit and to receive (J/bit) |
| \( U(\cdot) \) | Utility function |

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2. We note that \( Q(i) = D(i) \) for \( \beta_{\text{nonlin}} \to \infty \).

3. We note that the utility maximization framework achieves proportional fairness for \( U(\cdot) = \log(\cdot) \) and max-min fairness for \( U(\cdot) = (\cdot)^{1-\alpha}/(1 - \alpha) \) with \( \alpha \to \infty \) [36].
emulators. The traces are available at enhants.ee.columbia.edu and in the CRAWDAD repository [19].

### 4.1 Device Energy Budgets and Daily Energy Availability

Sample irradiance measurements (for three setups over the same 10 days) are provided in Fig. 5. One use of the measurements is to determine energy budgets for indoor energy harvesting devices. Hence, we calculate the total daily irradiation $H_d$, representing energy incident onto 1 cm$^2$ area over the entire course of a day. Fig. 6 demonstrates the $H_d$ values for setup L-1. Table 2 presents the average and the standard deviation values, $\overline{H}_d$ and $\sigma(H_d)$. These bit rates are calculated assuming solar cell efficiency of $\eta = 1\%$ (i.e., efficiency of an organic solar cell) and solar cell size $A = 10$ cm$^2$. We note that for the different setups, the $H_d$ values vary greatly. The differences are related to office layouts, presence or absence of direct sunlight, as well as the use of shading, windows, and indoor lights. Table 2 also shows the bit rate $r$ a node would be able to maintain throughout a day when exposed to irradiation $H_d$. As an energy cost to communicate, 1 nJ/bit is used [18]. These bit rates can be seen as “communication budgets” for light energy harvesting devices (such as EnHANTS [18], [46], [53]) deployed in indoor environments.

To predict daily energy availability $H_d$, a node can use a simple exponential smoothing approach, calculating a predictor for slot $i$, $\overline{H}_d(i)$, as $\overline{H}_d(1) \leftarrow \overline{H}_d(0)$, $\overline{H}_d(i) \leftarrow \alpha \cdot \overline{H}_d(i-1) + (1 - \alpha) \cdot H_d(i-1)$ for $\alpha$ constant, $0 \leq \alpha \leq 1$. The error for such a simple predictor is relatively high. For example, for setup L-1 the average prediction error is over $0.4\overline{H}_d$, and for setup L-2 it is over $0.5\overline{H}_d$. For the outdoor data sets, the average prediction errors are approximately $0.3\overline{H}_d$.

Improving the energy predictions for indoor conditions using weather forecasts has been studied in [31] and [45]. We examined whether the $H_d$ values in the indoor settings are correlated with the weather data [6]. We determined statistically significant correlations for all setups except L-2. This suggests that for some indoor setups the energy predictions may be improved, similar to outdoor environments, by incorporating the weather forecasts into the predictions.

Work week pattern also influences indoor radiant energy in office environments, particularly for setups that do not receive direct sunlight. For setup L-2, for example, $\overline{H}_d = 1.63$ J/cm$^2$ on weekdays, and $\overline{H}_d = 0.37$ J/cm$^2$ on weekends (it receives, on average, 9.7 hours of office lighting per day on weekdays and under 1 hour on weekends). By keeping separate predictors for weekends and weekdays, the average prediction error for the weekdays is lowered from $0.5\overline{H}_d$ to $0.26\overline{H}_d$.

We also examined correlations between the $H_d$ values of different data sets, and determined statistically significant correlations for a number of setups. For example, for setups L-1 and L-2 located in the same room, $\rho = 0.56$ ($p < 0.001$), and for setups L-1 and L-5 facing in the same direction, $\rho = 0.71$ ($p < 0.001$). This indicates that in a network of energy harvesting devices, a device will be able to infer some information about its peers’ energy availability based on its own (locally observed) energy state.

### 4.2 Short-Term Energy Profiles

To characterize energy availability at different times of day, we determine the $H_T$ values for different 0.5 hour intervals $T$, generating energy profiles for the setups. Sample energy profiles are shown in Fig. 7, where the left side shows the irradiance curves corresponding to different days overlayed on each other, and the right side shows the $\overline{H}_T$ values, with errorbars representing $\sigma(H_T)$. Due to variations in illumination and occupancy patterns, the energy profiles of different locations can be very different. For example, while setup L-3 exhibits daylight-dependent variations in irradiance, for setup L-2 the irradiance is either 0 or 45 μW/cm$^2$ for most of the day (as this setup receives mostly indoor light). In addition, while for setup L-2 the lights are often on during late evening hours, for setup L-3 it is almost never the case. The demonstrated $\sigma(H_T)$ values suggest that these energy inputs generally fall under the partially predictable profile energy models.

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#### TABLE 2

<table>
<thead>
<tr>
<th>Location index</th>
<th>Location description</th>
<th>Experiment timeline</th>
<th>$\overline{H}_d$ (J/cm$^2$/day)</th>
<th>$\sigma(H_d)$</th>
<th>$r$ (Kbs, cont.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-1</td>
<td>Students’ office. South-facing, 6th floor above ground, window-wall-located setup</td>
<td>Aug. 15, 2009 – Sept. 13, 2010</td>
<td>1.3</td>
<td>0.72</td>
<td>1.5</td>
</tr>
<tr>
<td>L-2</td>
<td>Students’ office (same office as setup L-1). Setup on a table far from windows, receiving direct sunlight for a short portion of a day</td>
<td>Nov. 13, 2009 – Sept. 9, 2010</td>
<td>1.28</td>
<td>0.76</td>
<td>1.5</td>
</tr>
<tr>
<td>L-3</td>
<td>Departmental conference room. North-facing, 13th floor above ground, with large unobstructed windows. Window-wall-located setup</td>
<td>Nov. 7, 2009 - Sept. 13, 2010</td>
<td>63.0</td>
<td>48.0</td>
<td>72.0</td>
</tr>
<tr>
<td>L-4</td>
<td>Students’ office. South-West-facing, window-wall-located setup</td>
<td>Nov. 5, 2009 – Sept. 29, 2010</td>
<td>9.2</td>
<td>6.9</td>
<td>7.9</td>
</tr>
<tr>
<td>L-5</td>
<td>Students’ office (directly under the office of setup L-1). Window-wall-located setup</td>
<td>June 25, 2009 – Oct. 11, 2010</td>
<td>12.3</td>
<td>8.3</td>
<td>13.9</td>
</tr>
<tr>
<td>L-6</td>
<td>Students’ office. East-facing, often receiving unattenuated reflected outdoor light. Window-wall-located setup</td>
<td>Feb. 15, 2010 – Sept. 20, 2010</td>
<td>97.3</td>
<td>64.4</td>
<td>112.3</td>
</tr>
<tr>
<td>O-1</td>
<td>Outdoor: ECSU metestation [3], Elizabeth City, NC.</td>
<td>Jan. 1, 2009 – Dec. 31, 2009</td>
<td>1517</td>
<td>787</td>
<td>1,750</td>
</tr>
</tbody>
</table>
setup before 08:00 AM, the correlation between the amount of energy received collected in a particular time slot.

We examined correlations between the amount of energy device has observed some of the incoming energy [7], [31].

(2 March 2010-12 March 2010).

We have also conducted shorter term experiments for

4.3 Mobile Measurements

Fig. 4. A schematic diagram of the indoor irradiance measurement locations L-1-L-6.

Fig. 5. Sample irradiance measurements in locations L-2, L-3, and O-1 (2 March 2010-12 March 2010).

We have studied whether, similarly to outdoor environments, in the indoor environments the accuracy of the energy profile for a given day can be improved when a device has observed some of the incoming energy [7], [31]. We examined correlations between the amount of energy collected in a particular time slot \( i \), \( H_T(i) \), and the amount of energy available in some later time slot \( j \), \( H_T(j) \) (where \( j > i \)). We also examined correlations between the amount of energy collected up to a particular time slot \( j \), \( \sum_{i<j} H_T(i) \), and the energy collected over the subsequent time slots, \( \sum_{i\geq j} H_T(i) \). We determined that such correlations are present in indoor environments, but are generally stronger in outdoor settings. For example, for the outdoor setup O-1 the correlation between the energy received in the 21st time slot (10:30-11:00 AM) and in the 33rd time slot (16:30-17:00 PM) is \( \rho = 0.5 \) \((p < .001)\), while for the indoor setup L-1 it is \( \rho = 0.2 \) \((p < .001)\). For the outdoor setup O-1, the correlation between the amount of energy received before 08:00 AM, \( \sum_{i<16} H_T(i) \), and the amount of energy received after 08:00 AM, \( \sum_{i\geq 16} H_T(i) \), is \( \rho = 0.77 \) \((p < .001)\), while for the indoor setup L-3 it is \( \rho = 0.31 \) \((p < .001)\). These results suggest that profile prediction techniques developed for outdoor systems may be extended to indoor environments, but their performance indoors is likely to be worse.

4.3 Mobile Measurements

We have also conducted shorter term experiments for mobile devices. Table 3 provides a summary of the measurements conducted, demonstrating average irradiance \( T \), standard deviation of the irradiance \( \sigma(T) \), and the corresponding sustainable bit rate \( r \). It can be observed that energy availability differs drastically for different experimental conditions.

A sample irradiance trace for a measurement setup carried around Times Square in New York City at nighttime (measurement set M-6) was shown in Fig. 1c. Fig. 8 demonstrates an irradiance trace of a device carried around a set of indoor and outdoor locations (note the log scale of the \( y \)-axis) during mid-day on a sunny day (measurements set M-1). These measurements highlight the disparity between the light energy available indoors and outdoors. For example, inside a lab, the irradiance was 70 \( \mu \text{W/cm}^2 \), while in sunny outdoor conditions it was 32 mW/cm\(^2\). Namely, the outdoor to indoor energy ratio was more than 450 times. In general, we observed that mobile devices’ energy levels are diverse, poorly predictable, and could in some cases be represented by stochastic energy models.

5 Deterministic Energy Profile

In this section, we consider the deterministic profile energy model (similar to the models studied in [12], [27], and [38]). We formulate optimization problems that apply to both linear and nonlinear energy storage \(^8\) for a single node and for a link, and introduce algorithms for solving the formulated problems.

5.1 Single Node: Optimizing Energy Spending

To achieve smooth energy spending for a node, we formulate the following problems where we optimize the node energy allocation vector \( \{s(i)\} \) using the utility maximization and lexicographic maximization frameworks.

Time Fair Utility Maximization (TFU) Problem:

\[
\max_{s(i)} \sum_{i=0}^{K-1} U(s(i)),
\]

s.t.: \( s(i) \leq B(i) \quad \forall \quad i \),

\( B(i) \leq B(i+1) + Q(i+1) - B(i) \quad \forall \quad i \geq 1 \),

\( B(i) \leq C \quad \forall \quad i \),

\( B(0) = B_0; \quad B(K) \geq B_K \),

\( B(i), s(i) \geq 0 \quad \forall \quad i \).

\(^8\) Recall that a linear energy storage model applies to a battery and that a nonlinear energy storage model may represent a capacitor.
Recall that $U(s(i))$ is a concave nondecreasing function. Recall, additionally, that for linear energy storage, $Q(i) = D(i)$, and for nonlinear energy storage, $Q(i) = q(D(i), B(i))$ (see Section 3). Constraint (3) ensures that a node does not spend more energy than it has stored, (4) and (5) represent the energy storage evolution dynamics, and (6) sets the initial and final energy storage levels to $B_0$ and $B_K$.

**Time Fair Lexicographic Assignment (TFLA) Problem:**

Lexicographically maximize: $\{s(0), \ldots, s(K-1)\}$ \hspace{1cm} (8)

s.t.: constraints (3)-(7).

Fig. 9 shows an example of node energy allocation vectors $\{s(i)\}$ obtained by solving the TFU and the TFLA problems. Fig. 9a shows the energy profile $\{D(i)\}$ used as an input to these problems. This energy profile corresponds to the light energy available in an indoor location (see Table 2). Fig. 9b shows the energy allocation vectors $\{s(i)\}$ obtained by solving the TFLA problem under the linear energy storage model and by solving the TFU problem under the nonlinear energy storage model.9

Next, we provide a general algorithm (for linear and nonlinear energy storage) of a relatively high complexity, a faster algorithm for linear energy storage, and a very fast algorithm for large linear energy storage.

Assuming energy inputs and energy storage to be quantized, the TFU problem can be solved by the dynamic programming-based Time Fair Rate Assignment (TFR) algorithm (Algorithm 1).10

**Algorithm 1 Time Fair Rate Assignment (TFR).**

$h(i, B) \leftarrow -\infty$, $s(i) \leftarrow 0$ $\forall i < K$, $\forall B$; $h(K, B) \leftarrow -\infty$ $\forall B < B_K$; $h(K, B) \leftarrow 0$ $\forall B \geq B_K$; for $i = K-1; i \geq 0; i \leftarrow i-1$; do

for $B = 0; B \leq C; B \leftarrow B + \Delta$; do

for $s = 0; s \leq B; s \leftarrow s + \Delta$; do

$\hat{s} \leftarrow s$; $h \leftarrow U(\hat{s}) + h(i+1, \min(B+q(D(i), B) - \hat{s}, C))$;

if $\hat{h} > h(i, B)$ then

$h(i, B) \leftarrow \hat{h}$; $s(i) \leftarrow \hat{s}$;

return $h(0, B_0)$, and associated $s(i)$ $\forall i$

In the TFR algorithm, for each $\{i, B(i)\}$ we determine $h(i, B(i)) = \max_{s(i) \leq B(i)} [U(s(i)) + h(i + 1, \min(B(i) + O(D(i), B) - s(i), C))].$

Going “backwards” from $i = K - 1$, we thus obtain a vector $\{s(0), \ldots, s(K-1)\}$ that maximizes $h(0, B_0)$; this is the optimal energy allocation vector. Recall that we denote the energy quantization resolution by $\Delta$. In the TFR algorithm, we calculate $h(i, B(i))$ for each of the $K \cdot (C/\Delta)$ tuples.

---

9. The solutions were obtained for the following parameters: $C = 0.5 \cdot Q$, $B_0 = B_K = 0.4 \cdot C$, $U(s(i)) = \log(s(i))$, and $\beta_{\text{min}} = 1.05$.

10. While other ways of solving the TFU problem can be considered, dynamic programming offers a natural solution approach.
each tuple \( \{i, B(i)\} \). Maximizing an instance of \( h(i, B(i)) \) requires considering all \( s(i) \) such that \( s(i) \leq B(i) \leq C \). Thus, for each tuple \( \{i, B(i)\} \), the TFR algorithm performs at most \( C/\Delta \) operations. The running time of the TFR algorithm is, therefore, \( O(K \cdot [C/\Delta]^2) \).

For linear energy storage (\( q(D(i), B(i)) = D(i) \), i.e., a battery), we refer to the TFU and the TFLA-LIN problems as TFU-LIN and TFLA-LIN. For these problems, we obtain the following Lemma, whose proof is given in Appendix I, which can be found on the Computer Society Digital Library at http://doi.ieeeecomputersociety.org/10.1109/TMC.2012.154.

**Lemma 1.** The optimal solutions to the TFU-LIN problem and the TFLA-LIN problem are equal.

For solving the TFU-LIN and the TFLA-LIN problems, we develop the Progressive Filling (PF) algorithm (Algorithm 2), inspired by the algorithms for max−min fair flow control [9]. The PF algorithm starts with \( s(i) = 0 \) ∀ \( i \), and iterates through the slots, increasing the \( s(i) \) value of each slot by \( \Delta \) on every iteration. The algorithm verifies that increasing \( s(i) \) does not result in shortage of energy for other slots, or in the lack of final energy \( B_i \). An \( s(i) \) value is increased only when it does not interfere with the spending in slots with smaller \( s(i) \) values, thus the resulting solution is max−min fair. At each step of the PF algorithm, the verification subroutine of complexity \( O(K) \) is executed. Recall that \( Q = \sum_i Q(i) + (B_0 - B_K) \). The algorithm takes \( Q/\Delta \) spending increase steps, and \( K \) additional steps to “fix” the slots. Thus, the PF algorithm runs in \( O(K \cdot [K + Q/\Delta]) \) time. Assuming that \( K \) is small compared to \( Q/\Delta \), for \( C \) and \( Q \) that are on the same order, the PF algorithm is faster than the TFR algorithm.

**Algorithm 2 Progressive Filling (PF).**

\[
A_{fix} \leftarrow \emptyset; \quad s(i) \leftarrow 0 \forall i; \\
\text{while } A_{fix} \neq \emptyset \text{ do} \\
\quad \text{for } i = 0; i \leq K - 1; i + + \text{ do} \\
\quad \quad \text{if } i \notin A_{fix} \text{ then} \\
\quad \quad \quad \tilde{s}(j) \leftarrow s(j) \forall j \in [0, K - 1]; \tilde{s}(i) \leftarrow \tilde{s}(i) + \Delta; \\
\quad \quad \quad \text{valid} \leftarrow \text{check_validity}(\tilde{s}); \\
\quad \quad \quad \text{if } \text{valid} = \text{TRUE} \text{ then } s(i) \leftarrow \tilde{s}(i); \\
\quad \quad \quad \text{else } A_{fix} := A_{fix} \cup i; \\
\quad \quad \text{function check_validity}(\tilde{s}): \\
\quad \quad \quad B(i) \leftarrow 0 \forall i; B(0) \leftarrow B_0; \text{valid} \leftarrow \text{TRUE}; \\
\quad \quad \quad \text{for } i = 1; i \leq K; i + + \text{ do} \\
\quad \quad \quad \quad B(i) \leftarrow \min(C, B(i - 1) + Q(i - 1) - \tilde{s}(i - 1)); \\
\quad \quad \quad \quad \text{if } \tilde{s}(i) > B(i) \text{ then } \text{valid} \leftarrow \text{FALSE}; \\
\quad \quad \quad \quad \text{if } B(K) < B_K \text{ then } \text{valid} \leftarrow \text{FALSE}; \\
\quad \quad \quad \text{return } \text{valid}
\]

Finally, when the energy storage is large compared to the energy harvested, the TFLA-LIN and TFU-LIN problems can be solved easily. Below we define Large Storage (LS) and generalized Large Storage (LS-gen) Conditions, and demonstrate that when they hold, the optimal policy is a simple one.\(^{11}\) Let \( s(i) = Q/K \forall i \), and let \( \tilde{B}(i) = \sum_{j=0}^{i} Q(j) - (i - 1) \cdot s(i) \forall 1 \leq i \leq K \).

**Definition 1.** The LS Conditions hold, if

\[
B_0 \geq \min_{1 \leq i \leq K} \tilde{B}(i)
\]

and

\[
C - B_0 \geq \max_{1 \leq i \leq K} \tilde{B}(i).
\]

**Definition 2.** The LS-gen Conditions hold, if \( B_0 \geq \sum_i Q(i) \cdot (1 - 1/K) \) and \( C - B_0 \geq \sum_i Q(i) \cdot (1 - 1/K) \).

The proofs of the following lemmas are given in Appendices II and III, which are available in the online supplemental material, respectively.

**Lemma 2.** When the LS Conditions or the LS-gen Conditions hold, the optimal solution to the TFLA-LIN is \( s(i) = Q/K \forall i \).

**Lemma 3.** When the LS Conditions or the LS-gen Conditions hold, the optimal solution to the TFU-LIN, for \( U(s(i)) \) that are twice differential strictly concave on \( (0, \hat{Q}) \), and that satisfy

\[
(I) \quad U''(0) > 0 \quad \text{on} \quad (0, \hat{Q}) \quad \text{and} \quad U''(\hat{Q}) = 0, \\
(II) \quad U''(0) > 0 \quad \text{on} \quad (0, \hat{Q}), \\
(III) \quad U''(0) > 0 \quad \text{on} \quad (0, \hat{Q}) \quad \text{and} \quad \lim_{x \to 0} U(x) = -\infty,
\]

is \( s(i) = \hat{Q}/K \forall i \).

Examples of \( U(\cdot) \) that satisfy (I), (II), and (III) include: (I): \( U(\cdot) = (\cdot)^{1-\alpha}/(1 - \alpha) \) for \( 0 < \alpha < 1 \) [36], (II): \( U(\cdot) = \log(\alpha + \cdot) \) for \( \alpha > 0 \), used, for \( \alpha = 1 \), in, for example, [11], [14], and (III): \( U(\cdot) = \log(\cdot) \), used in, for example, [34]. Verifying that the LS Conditions (or the LS-gen Conditions) hold and determining the corresponding optimal policy is computationally inexpensive.

### 5.2 Link: Optimizing Data Rates

For a link, we formulate the following problems where we optimize data rate allocation vectors \( \{r_u(i), r_v(i)\} \).

**Link Time Fair Utility Maximization (LTFU) problem:**

\[
\max_{r_u(i), r_v(i)} \sum_{i=0}^{K-1} \left[ U(r_u(i)) + U(r_v(i)) \right], \\
\text{s.t. : } \quad c_{tx} r_u(i) + c_{tx} r_v(i) \leq s_u(i) \forall i, \\
\quad c_{tx} r_u(i) + c_{tx} r_v(i) \leq s_v(i) \forall i,
\]

\[u, v: \text{constraints (3)-(7)}.
\]

\(^{11}\) To determine if the LS Conditions hold, a node needs to know \( \{Q(0), \ldots, Q(K-1)\} \), while determining if the LS-gen Conditions hold requires only the knowledge of \( \sum Q(i) \). LS-gen Conditions can be used, for example, if light energy harvesting nodes characterize their energy availability by the daily irradiation \( H_i \) and do not calculate their energy profiles (see Section 4.2).
Fig. 10. (a) Energy profiles of link endpoints $u$ and $v$, and (b) the corresponding data rate allocation vectors $\{r_u(i)\}$ and $\{r_v(i)\}$ obtained by solving the LTFL and the LTFU problems.

**Link Time Fair Lexicographic Assignment (LTFL) problem:**

Lexicographically maximize:

$$\{r_u(0), \ldots, r_u(K-1), r_v(0), \ldots, r_v(K-1)\}$$ (12)

s.t.: $(10), (11)$; $u, v$: constraints $(3)-(7)$.

Since the optimal solution to the LTFL problem is $\max - \min$ fair, it assigns the data rates such that $r_u(i) = r_v(i) \forall i$ (since for the $\max - \min$ fairness objective no increase in one of the rates can “outweigh” the decrease in the other). Thus, the LTFL problem can be restated as:

Lexicographically maximize: $\{r(0), \ldots, r(K-1)\}$,

$$\text{s.t.: } r(i) \cdot (c_{tx} + c_{rx}) \leq \min(s_u(i), s_v(i)) \forall i,$$ (14)

$$u, v: \text{constraints (3)-(7)},$$

where $r(i) = r_u(i) = r_v(i)$.

Examples of solutions to the LTFU and LTFL problems are shown in Fig. 10. Fig. 10a shows the energy profiles of nodes $u$ and $v$. These energy profiles correspond to the light energy available in indoor locations L-1 and L-2 (see Table 2) on the same day. Fig. 10b shows the data rate allocation vectors $\{r_u(i)\}$ and $\{r_v(i)\}$ obtained by solving the LTFU and the LTFL problems.

In general, the solutions to the LTFL and LTFU problems are not the same. The following observation, whose proof appears in Appendix IV, which is available in the online supplemental material, identifies a case where the solutions are identical.

**Observation 1.** When $c_{tx} = c_{rx}$, the LTFL problem and the LTFU problem have the same solution.

For quantized energy values, the LTFU problem can be solved with an extension of the TFR algorithm, referred to as LTFR. Over all $\{r_u(i), r_v(i)\}$ such that $c_{tx}r_u(i) + c_{rx}r_v(i) = s_u(i) \leq B_u(i), c_{tx}r_u(i) + c_{rx}r_v(i) = s_v(i) \leq B_v(i)$, the LTFR algorithm determines, for each $i$, $B_u(i), B_v(i)$,

$$h(i, B_u(i), B_v(i)) = \max[U(r_u(i)) + U(r_v(i))] + h(i + 1),$$

$$\min[B_u(i) + Q_u(i) - s_u(i), C_u],$$

$$\min[B_v(i) + Q_v(i) - s_v(i), C_v]].$$

Vectors $\{r_u(0), \ldots, r_u(K-1)\}$ and $\{r_v(0), \ldots, r_v(K-1)\}$ that maximize $h(0, B_{u,0}, B_{v,0})$ are the optimal. Since this formulation considers all $\{i, B_u(i), B_v(i)\}$ combinations and examines all feasible rates $r_u(i)$ and $r_v(i)$ for each combination, the overall complexity of the LTFR algorithm is $O(K \cdot \left[ C_u / \Delta \right]^2 \cdot \left[ C_v / \Delta \right]^2)$.

For linear energy storage, the LTFL problem can be solved by an extension of the PF algorithm, referred to as the LPF algorithm. Similarly to the PF algorithm, the LPF algorithm goes through all slots and increases the slots’ allocation by $\Delta$ when an increase is feasible. Unlike the PF algorithm, however, the LPF algorithm allocates the energy of both nodes $u$ and $v$. The running time of the LPF algorithm is $O(K \cdot [K + (Q_u + Q_v) / \Delta])$.

Solving the LTFU or the LTFL problems directly may be computationally taxing for small devices with limited capabilities. Instead, the nodes may use the following low complexity heuristic algorithms, which do not require extensive exchange of information.

**Decoupled Rate Control (DRC) algorithms.** Initially, nodes $u$ and $v$ determine independently from each other their energy spending rates $s_u(i)$ and $s_v(i)$ for every slot $i$ (i.e., using the PF algorithm). Then, for each slot $i$, under constraints (10) and (11), the nodes obtain a solution to

$$\max_{r_u(i), r_v(i)} U(r_u(i)) + U(r_v(i)) \tag{15}$$

if the LTFU problem is being solved (LTFU-DRC algorithm), and to $\max r(i)$ if the LTFL problem is being solved (LTFL-DRC algorithm). These subproblems (each considers a single slot $i$) can be easily solved. For the LTFL-DRC algorithm, due to (14), the subproblem solution is $r(i) = \min(s_u(i), s_v(i)) / (c_{tx} + c_{rx})$. For the LTFU-DRC algorithm, a closed-form $O(1)$ solution to the subproblem can be obtained for each particular function $U(s(i))$. For example, for $U(s(i)) = \log(s(i))$ with $c_{tx} = \rho c_{rx}, \rho > 0$ [18], for the case of $s_u(i) = \gamma s_v(i), 0 \leq \gamma \leq 1$, the optimal solution is either $\{r_u(i), r_v(i)\} = \{s_u(i) / (c_{tx} + \rho c_{rx}) \}$ or $\{r_u(i), r_v(i)\} = \{s_v(i) / (2 \cdot c_{tx})\}$. For linear energy storage, when the storage is large compared to the energy harvested for both $u$ and $v$, solving a single instance of the LTFU-DRC or LTFL-DRC problem obtains the overall solution. Moreover, as shown in the lemma below, in this case the DRC solution is optimal. Thus, in such case the optimal solution can be calculated with little computational complexity. The proofs of the following lemmas are given in Appendices V and VI, which are available in the online supplemental material, respectively.

**Lemma 4.** If the LS Conditions or the LS-gen Conditions hold for nodes $u$ and $v$, the LTFL-DRC algorithm obtains the optimal solution to the LTFL problem.

**Lemma 5.** If the LS Conditions or the LS-gen Conditions hold for nodes $u$ and $v$, for $U(\cdot)$ that are twice differential strictly concave on $(0, R]$, where $R = \max\{Q_u, Q_v\} / \min\{c_{tx}, c_{rx}\}$, and that satisfy

$$(I) \quad U'(0) > 0 \text{ on } (0, R] \text{ and } U'(0) = 0,$$

or $$(II) \quad U'(0) > 0 \text{ on } [0, R],$$

or $$(III) \quad U'(0) > 0 \text{ on } (0, R] \text{ and } \lim_{x \to 0} U(x) = -\infty,$$

where $C_u = 0.5 \cdot Q_u, B_{u,0} = B_{t,u} = B_{K,u} = B_{K,x} = 0.25 \cdot C_u, c_{tx} = 0.1 \text{ nJ/bit},$ $c_{rx} = 1 \text{ nJ/bit}$ [18], $U(r(i)) = \log(r(i))$, and $Q(i) = D(i)$ (linear energy storage)
6 STOCHASTIC ENERGY MODELS

In this section, we study models in which the energy harvested in a slot is an i.i.d. random variable \( D \). For tractability, we assume that \( D \) takes one of \( M \) discrete values \([d_1, \ldots, d_M]\) with probability \([p_1, \ldots, p_M]\). \( D \) may represent, for example, the energy harvested by a mobile device in a short (seconds or minutes) time slot. For time slots of days, it may represent the daily irradiation \( H_d \) received by a device (when the energy storage is relatively large, variations in energy availability within a day may be abstracted, and \( H_d \) can be used to characterize energy availability). We formulate the control problems and determine corresponding policies for a single node and for a link. The formulations apply to linear and nonlinear (e.g., a capacitor) energy storage models. For a given distribution of \( D \), the optimal policy needs to be calculated once. Thus, operating according to the optimal policy does not require frequent computations.

**Spending Policy Determination (SPD) problem.** For a given distribution of \( D \), determine the energy spending rates \( s(i) \) such that

\[
\max_{s(i)} \lim_{K \to \infty} \frac{1}{K} \sum_{i=0}^{K-1} U(s(i)).
\]

This discrete time stochastic control process is an average cost MDP, and can be solved with standard MDP solution techniques. For example, using value iteration approach and applying dynamic programming, we consider a large number of slots \( K \), and going “backwards” from \( i = K - 1 \), for each \( \{i, B(i)\} \), determine

\[
h(i, B(i)) = \max_{s(i) \leq B(i)} \mathbb{E}[U(s(i))
+ h(i + 1, \min[B(i) + q(D(i), B(i)) - s(i), C])]
= \max_{s(i) \leq B(i)} \left[ U(s(i)) + \sum_{j=1}^{M} p_d j \cdot h(i + 1,
\min[B(i) + q(d_j, B(i)) - s(i), C]) \right].
\]

(16)

Performing this iterative procedure for a large number of slots \( K \), we obtain, for each energy storage level \( B(i) \), a corresponding stationary (same for all values of \( i \)) \( s(i) \) value that approaches the optimal [22]. Although policy calculations are computationally expensive (the running time of this algorithm is \( O(|C|/\Delta^2 \cdot M \cdot K) \)), such a policy needs to be computed only once for a particular distribution of \( D \). Fig. 11 presents example optimal energy spending policies obtained by solving the SPD problem for linear and nonlinear energy storage models. The daily irradiation \( H_d \) for setup L-1 (see Fig. 6) is used as the random variable \( D \).

For a link, we define the following problem.

**Link Spending Policy Determination (LSPD) Problem:**

\[
\max_{r_u(i), r_v(i)} \lim_{K \to \infty} \frac{1}{K} \sum_{i=0}^{K-1} [U(r_u(i)) + U(r_v(i))].
\]

(17)

Similarly to the SPD problem, the LSPD problem can be solved with standard approaches to solving MDPs. For example, using value iteration approach, we determine, for each \( \{i, B_u(i), B_v(i)\} \),

\[
h(i, B_u(i), B_v(i)) = \max_{D_u, D_v} \mathbb{E}[U(r_u(i)) + U(r_v(i))
+ h(i + 1, \min[B_u(i) + q(D_u(i), B_u(i))
- s_u(i), C_u], \min[B_v(i) + q(D_v(i), B_v(i))
- s_v(i), C_v]),
\]

(18)

where the maximization is over all \( \{r_u(i), r_v(i)\} \) such that \( c_u r_u(i) + c_v r_v(i) = s_u(i) \leq B_u(i) \), \( c_u r_u(i) + c_v r_v(i) = s_v(i) \leq B_v(i) \). This procedure is computationally complex. Similarly to the SPD problem, it needs to be solved for a large number of slots \( K \), and has the complexity \( O(|C_u|/\Delta^2 \cdot |C_v|/\Delta^2 \cdot M_u \cdot M_v \cdot K) \). However, it needs to be computed only once.

Fig. 12 demonstrates example optimal link rate assignment policy \( \{r_u(i), r_v(i)\} \) as a function of \( \{B_u(i), B_v(i)\} \) obtained by solving the LSPD problem. The daily irradiation \( H_d \) for

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13. The solutions were obtained for the following parameters: \( C = 2.7 \cdot \mathbb{E}(D(i)), U(s(i)) = \log(1 + s(i)) \), and \( \lambda_{\text{min}} = 1.3 \).
setup L-1 (see Fig. 6) is used as the random variable $D_u$ and the random variable $D_v$. The MDP formulations can be easily extended to consider other parameters, such as the cost to change the energy spending rates $s(i)$, or the cost to change data rates $r_u(i)$ and $r_v(i)$.

### 7 Numerical Results

This section provides numerical results demonstrating the use of the algorithms described in Section 5. Measurement traces described in Section 4 are used as inputs to the algorithms.

Fig. 13 shows the optimal energy spending allocation vectors $\{s(i)\}$ for the TFU and the TFLA problems presented in Section 5.1, for different values of energy storage capacity $C$ and initial energy storage state $B_0$.\(^{14}\) The energy profile $\{D(i)\}$ used as an input to these algorithms is shown in Fig. 13a. It corresponds to the average daily energy profile for the indoor location L-3 (see Table 2). Fig. 13b demonstrates the energy spending rate allocations $\{s(i)\}$ that solve the TFLA-LIN and the TFU-LIN problems (that is, linear energy storage model). These spending rates were obtained using the PF algorithm. It can be observed that larger energy storage allows for “smoother” energy allocation. For this energy profile $\{D(i)\}$, the LS Conditions described in Section 5 are matched when $C = 2$ J and $B_0 = 1$ J. It can be observed that in this case the energy spending rate allocation vector $\{s(i)\}$ corresponds to the optimal policy given by Lemmas 2 and 3. Fig. 13c shows the optimal solutions of the TFU problem with nonlinear energy storage (for $\beta_{\text{nonlin}} = 1.1$) obtained using the TFR algorithm. Such a system has not been analyzed before.

Fig. 14 shows the numerical results for the link data rate determination problems presented in Section 5.2. The energy profiles of indoor setups L-1 and L-2 (see Fig. 10a) were used as inputs to the algorithms. The optimal solutions to the LTFL and the LTU problems for linear energy storage model have been shown in Fig. 10. Fig. 14a shows the optimal solution to the LTU problem for nonlinear energy storage (for $\beta_{\text{nonlin}} = 1.1$) obtained using the LTFR algorithm. Fig. 14b shows the communication rate assignment vectors $\{r_u(i)\}$ and $\{r_v(i)\}$ calculated using a simple LTU-DRC algorithm for linear energy storage. In this example, the LTU-DRC algorithm obtains data rate assignments $\{r_u(i), r_v(i)\}$ that are similar to those obtained by optimally solving the LTU-LIN problem.

### 8 Conclusions and Future Work

Motivated by recent advances in the areas of energy harvesting and ultra-low-power communications, in this work, we focus on energy harvesting devices. We described the first long-term indoor radiant energy measurements campaign that provides useful energy traces, as well as insights into the design of systems and algorithms. We developed algorithms for deterministic environments that uniquely determine the energy management and data rate control policies for linear and nonlinear energy storage models, for single node and node pair (link) scenarios. The algorithms for the predictable case also provide insight into the partially predictable case. We developed algorithms for stochastic environments that can provide nodes with simple pre-computed decision policies. We used the algorithms to obtain numerical results for various cases.

We covered a few “working points” in the design space described in Section 1. Yet, there are still many other working points to study. In particular, although some algorithms have been developed for networks of nodes, most of them are too complex for resource-constrained nodes. In our ongoing work, we are analyzing the performance of simple policies for energy harvesting devices for single node and link cases [14]. We plan to develop simple energy-harvesting-aware algorithms for networks of nodes, additionally considering various other problem dimensions. Moreover, we plan to evaluate these algorithms in an energy harvesting active networked tags (EnHANTs) testbed that we are currently building [46], [53].

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